Generic and Flexible Defaults for Verified,Law-Abiding Type-Class Instances

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How can we make verifying type class laws more pleasant?

```haskell
class Ord a where
  (<=) :: a -> a -> Bool
```

```haskell
-- Transitivity:
--   if x <= y && y <= z,
--   then x <= z
class Ord a where
  (<=) :: a -> a -> Bool
```

```haskell
class Ord a => VOrd a where
  leqTransitive
    :: Π (x, y, z :: a)
    -> (x <= y) :~: True
    -> (y <= z) :~: True
    -> (x <= z) :~: True
```

```
class Ord a => VOrd a where
  leqTransitive
    :: Π (x, y, z :: a)
    -> (x <= y) :~: True
                    True
                    True
```

```
data a :~: b where
  Refl :: a :~: a
```

```
class Ord a => VOrd a where
  leqTransitive
    :: Π (x, y, z :: a)
    -> (x <= y) :~: True
    -> (y <= z) :~: True
    -> (x <= z) :~: True
```

Writing out proofs can be tiresome

```
data T a = MkT1 a
```

```haskell
data T a = MkT1 a

instance Ord a => Ord (T a) where
  (MkT1 x) <= (MkT1 y) = (x <= y)
```

```haskell
data T a = MkT1 a

instance Ord a => Ord (T a) where
  (MkT1 x) <= (MkT1 y) = (x <= y)

instance VOrd a => VOrd (T a) where
  leqTransitive (MkT1 x) (MkT1 y) (MkT1 z) Refl Refl
    | Refl <- leqTransitive x y z Refl Refl
    = Refl
```

```haskell
data T a = MkT1 a

instance Ord a => Ord (T a) where
  (MkT1 x) <= (MkT1 y) = (x <= y)

instance VOrd a => VOrd (T a) where
  leqTransitive (MkT1 x) (MkT1 y) (MkT1 z) Refl Refl
    | Refl <- leqTransitive x y z Refl Refl
    = Refl
```

```haskell
data T a = MkT1 a | MkT2 a

instance Ord a => Ord (T a) where
  (MkT1 x) <= (MkT1 y) = (x <= y)

instance VOrd a => VOrd (T a) where
  leqTransitive (MkT1 x) (MkT1 y) (MkT1 z) Refl Refl
    | Refl <- leqTransitive x y z Refl Refl
    = Refl
```

```haskell
data T a = MkT1 a | MkT2 a

instance Ord a => Ord (T a) where
  (MkT1 x) <= (MkT1 y) = (x <= y)
  (MkT2 x) <= (MkT2 y) = (x <= y)
  (MkT1 _) <= (MkT2 _) = True
  (MkT2 _) <= (MkT1 _) = False
```

```haskell
data T a = MkT1 a | MkT2 a

instance VOrd a => VOrd (T a) where
  leqTransitive t t' t'' Refl Refl =
    case (t, t', t'') of
      (MkT1 x, MkT1 y, MkT1 z)
        | Refl <- leqTransitive x y z Refl Refl
        = Refl
      (MkT2 x, MkT2 y, MkT2 z)
        | Refl <- leqTransitive x y z Refl Refl
        = Refl
      (MkT1 _, _, MkT2 _)
        = Refl
```

```
data T a = MkT1 a | MkT2 a

instance VOrd a => VOrd (T a) where
  leqTransitive t t' t'' Refl Refl =
    case (t, t', t'') of
      (MkT1 x, MkT1 y, MkT1 z)
        | Refl <- leqTransitive x y z Refl Refl
        = Refl
      (MkT2 x, MkT2 y, MkT2 z)
        | Refl <- leqTransitive x y z Refl Refl
        = Refl
      (MkT1 _, _, MkT2 _)
        = Refl
```

```haskell
data T a = MkT1 a | MkT2 a | MkT3 a
  instance VOrd a => VOrd (T a) where
    leqTransitive t t' t'' Refl Refl =
      case (t, t', t'') of
        (MkT1 x, MkT1 y, MkT1 z)
          | Refl <- leqTransitive x y z Refl Refl
          = Refl
        (MkT2 x, MkT2 y, MkT2 z)
          | Refl <- leqTransitive x y z Refl Refl
          = Refl
        (MkT1 _, _, MkT2 _)
          = Refl
```

```haskell
data T a = MkT1 a | MkT2 a | MkT3 a
```

```haskell
instance VOrd a => VOrd (T a) where
  leqTransitive t t' t'' Refl Refl =
    case (t, t', t'') of
      (MkT1 x, MkT1 y, MkT1 z)
        | Refl <- leqTransitive x y z Refl Refl
        = Refl
      (MkT2 x, MkT2 y, MkT2 z)
        | Refl <- leqTransitive x y z Refl Refl
        = Refl
      (MkT3 x, MkT3 y, MkT3 z)
        | Refl <- leqTransitive x y z Refl Refl
        = Refl
      (MkT1 _, _, MkT2 _)
        = Refl
      (MkT1 _, _, MkT3 _)
        = Refl
      (MkT2 _, _, MkT3 _)
        = Refl
```

```
data T a = MkT1 a | MkT2 a | MkT3 a
```

```
instance VOrd a => VOrd (T a) where
  leqTransitive t t' t'' Refl Refl =
    case (t, t', t'') of
      (MkT1 x, MkT1 y, MkT1 z)
        | Refl <- leqTransitive x y z Refl Refl
        = Refl
      (MkT2 x, MkT2 y, MkT2 z)
        | Refl <- leqTransitive x y z Refl Refl
        = Refl
      (MkT3 x, MkT3 y, MkT3 z)
        | Refl <- leqTransitive x y z Refl Refl
        = Refl
      (MkT1 _, _, MkT2 _)
        = Refl
      (MkT1 _, _, MkT3 _)
        = Refl
      (MkT2 _, _, MkT3 _)
        = Refl
```

```
data T a = MkT1 a | ... | MkTn a

  instance VOrd a => VOrd (T a) where
    leqTransitive t t' t'' Refl Refl =
      case (t, t', t'') of
        (MkT1 x, MkT1 y, MkT1 z)
          | Refl <- leqTransitive x y z Refl Refl
          = Refl
        (MkT2 x, MkT2 y, MkT2 z)
          | Refl <- leqTransitive x y z Refl Refl
          = Refl
        (MkT3 x, MkT3 y, MkT3 z)
          | Refl <- leqTransitive x y z Refl Refl
          = Refl
        (MkT1 _, _, MkT2 _)
          = Refl
        (MkT1 _, _, MkT3 _)
          = Refl
        (MkT2 _, _, MkT3 _)
          = Refl
```

```
data T a = MkT1 a | ... | MkTn a
```

```
instance VOrd a => VOrd (T a) where
  leqTransitive t t' t'' Refl Refl =
    case (t, t', t'') of
      (MkT1 x, MkT1 y, MkT1 z)
        | Refl <- leqTransitive x y z Refl Refl
        = Refl
      (MkT2 x, MkT2 y, MkT2 z)
        | Refl <- leqTransitive x y z Refl Refl
        = Refl
      (MkT3 x, MkT3 y, MkT3 z)
        | Refl <- leqTransitive x y z Refl Refl
        = Refl
      (MkT1 _, _, MkT2 _)
        = Refl
      (MkT1 _, _, MkT3 _)
        = Refl
      (MkT2 _, _, MkT3 _)
        = Refl
```

$\frac{n^2+n}{2}$ **cases!**

Our solution: datatype-generic proofs

Our solution: datatype-generic proofs (à la `GHC.Generics`)

How to deal with proof boilerplate

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Tactics

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Tactics

???

```
class Ord a => VOrd a where
  leqTransitive
    :: Π (x, y, z :: a)
    -> (x <= y) :~: True
    -> (y <= z) :~: True
    -> (x <= z) :~: True
```

```
class Ord a => VOrd a where
  leqTransitive
    :: Π (x, y, z :: a)
    -> (x <= y) :~: True
    -> (y <= z) :~: True
    -> (x <= z) :~: True
```

```haskell
class Ord a => VOrd a where
  leqTransitive
    :: Sing (x :: a)
    -> Sing (y :: a)
    -> Sing (z :: a)
    -> (x <= y) :~: True
    -> (y <= z) :~: True
    -> (x <= z) :~: True
```

```haskell
class Ord a => VOrd a where
  leqTransitive
    :: Sing (x :: a)
    -> Sing (y :: a)
    -> Sing (z :: a)
    -> (x <= y) :~: True
    -> (y <= z) :~: True
    -> (x <= z) :~: True
```

Uses the *singletons* encoding to emulate dependent types

```haskell
class Ord a => VOrd a where
  leqTransitive
    :: Sing (x :: a)
    -> Sing (y :: a)
    -> Sing (z :: a)
    -> (x <= y) :~: True
    -> (y <= z) :~: True
    -> (x <= z) :~: True
```

Uses the *singletons* encoding to emulate dependent types

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class Ord a => VOrd a where
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    :: Sing (x :: a)
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    -> (x <= y) :~: True
    -> (y <= z) :~: True
    -> (x <= z) :~: True
```

Uses the *singletons* encoding to emulate dependent types

```
class Ord a => VOrd a where
  leqTransitive
    :: Sing (x :: a)
    -> Sing (y :: a)
    -> Sing (z ::
    -> (x <= y) :
    -> (y <= z) :~: True
    -> (x <= z) :~: True
```

Uses the *singletons* encoding to emulate dependent types

```
type family x <= y
```

```haskell
class Ord a => VOrd a where
  leqTransitive
    :: Π (x, y, z :: a)
    -> (x <= y) :~: True
    -> (y <= z) :~: True
    -> (x <= z) :~: True
```

Our solution: datatype-generic proofs (à la `GHC.Generics`)

Our solution: datatype-generic proofs (à la `GHC.Generics`)

```haskell
instance Ord a => Ord (T a) where
  (MkT1 x) <= (MkT1 y) = (x <= y)
  (MkT2 x) <= (MkT2 y) = (x <= y)
  (MkT1 _) <= (MkT2 _) = True
  (MkT2 _) <= (MkT1 _) = False
```

```haskell
instance Ord a => Ord (T a) where
  (<=) = defaultLeq
```

```haskell
instance Ord a => Ord (T a) where
  (<=) = defaultLeq
instance Ord Bool where
  (<=) = defaultLeq
```

```haskell
instance Ord a => Ord (T a) where
  (<=) = defaultLeq
instance Ord Bool where
  (<=) = defaultLeq
instance Ord a => Ord (Maybe a) where
  (<=) = defaultLeq
```

```haskell
instance Ord a => Ord (T a) where
  (<=) = defaultLeq
instance Ord Bool where
  (<=) = defaultLeq
instance Ord a => Ord (Maybe a) where
  (<=) = defaultLeq
instance Ord a => Ord [a] where
  (<=) = defaultLeq
```

```haskell
defaultLeq :: (Generic a, Ord (Rep a))
           => a -> a -> Bool
defaultLeq x y = (from x <= from y)
```

```haskell
defaultLeq :: (Generic a, Ord (Rep a))
           => a -> a -> Bool
defaultLeq x y = (from x <= from y)
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defaultLeq :: (Generic a, Ord (Rep a))
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defaultLeq x y = (from x <= from y)
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```haskell
defaultLeq :: (Generic a, Ord (Rep a))
           => a -> a -> Bool
defaultLeq x y = (from x <= from y)
```

```haskell
class Generic a where
  type Rep a
  from :: a -> Rep a
  to   :: Rep a -> a
```

```haskell
defaultLeq :: (Generic a, Ord (Rep a))
           => a -> a -> Bool
defaultLeq x y = (from x <= from y)
```

```haskell
class Generic a where
  type Rep a
  from :: a -> Rep a
  to   :: Rep a -> a
```

1. **A canonical representation type (Rep)**

```haskell
defaultLeq :: (Generic a, Ord (Rep a))
           => a -> a -> Bool
defaultLeq x y = (from x <= from y)
```

```haskell
class Generic a where
  type Rep a
  from :: a -> Rep a
  to   :: Rep a -> a
```

1. A canonical representation type (Rep)
2. An isomorphism between a and Rep a

```haskell
defaultLeq :: (Generic a, Ord (Rep a))
           => a -> a -> Bool
defaultLeq x y = (from x <= from y)
```

```haskell
class Generic a where
  type Rep a
  from :: a -> Rep a
  to   :: Rep a -> a
```

1. A canonical representation type (Rep)
2. An isomorphism between a and Rep a
3. Generic instances can be derived automatically.

4. Rep is minimalist

Unit	`data U1 = MkU1`
Constants	`newtype K1 c = MkK1 c`
Products	`data a :*: b = a :**: b`
Sums	`data a :+: b = L1 a | R1 b`

4. Rep is minimalist

Unit	`data U1        = MkU1`
Constants	`newtype K1 c = MkK1 c`
Products	`data a :*: b = a :**: b`
Sums	`data a :+: b = L1 a | R1 b`

Every instance of Rep is some combination of these four types.

4. Rep is minimalist

Unit	`instance Ord U1`
Constants	`instance Ord c => Ord (K1 c)`
Products	`instance (Ord a, Ord b) => Ord (a :*: b)`
Sums	`instance (Ord a, Ord b) => Ord (a :+: b)`

Every instance of Rep is some combination of these four types.

Our solution: **datatype-generic** proofs (à la `GHC.Generics`)

Our solution:
datatype-generic proofs
(à la `GHC.Generics`)

```haskell
instance Ord a => Ord (T a) where
  (<=) = defaultLeq
instance Ord Bool where
  (<=) = defaultLeq
instance Ord a => Ord (Maybe a) where
  (<=) = defaultLeq
instance Ord a => Ord [a] where
  (<=) = defaultLeq
```

```haskell
instance VOrd a => VOrd (T a) where
  leqTransitive = defaultLeqTransitive
instance VOrd Bool where
  leqTransitive = defaultLeqTransitive
instance VOrd a => VOrd (Maybe a) where
  leqTransitive = defaultLeqTransitive
instance VOrd a => VOrd [a] where
  leqTransitive = defaultLeqTransitive
```

```
defaultLeqTransitive
  :: ???
  => Π (x, y, z :: a)  -> (x <= y) :~: True
  -> (y <= z) :~: True -> (x <= z) :~: True
defaultLeqTransitive = ???
```

```
defaultLeqTransitive
  :: (Generic a, VOrd (Rep a))
  => Π (x, y, z :: a)  -> (x <= y) :~: True
  -> (y <= z) :~: True -> (x <= z) :~: True
defaultLeqTransitive = ???
```

Unit	`instance VOrd U1`
Constants	`instance VOrd c => VOrd (K1 c)`
Products	`instance (VOrd a, VOrd b) => VOrd (a :*: b)`
Sums	`instance (VOrd a, VOrd b) => VOrd (a :+: b)`

Unit	`instance VOrd U1`
Constants	`instance VOrd c => VOrd (K1 c)`
Products	`instance (VOrd a, VOrd b) => VOrd (a :*: b)`
Sums	`instance (VOrd a, VOrd b) => VOrd (a :+: b)`

```
Unit         instance VOrd U1
Constants    instance VOrd c => VOrd (K1 c)
Products     instance (VOrd a, VOrd b) => VOrd (a :*: b)
Sums         instance (VOrd a, VOrd b) => VOrd (a :+: b)
                 where
               leqTransitive s s’ s’’ Refl Refl =
                 case (s, s’, s’’) of
                   (L1 x, L1 y, L1 z)
                     | Refl <- leqTransitive x y z
                                 Refl Refl = Refl
                   (R1 x, R1 y, R1 z)
                     | Refl <- leqTransitive x y z
                                 Refl Refl = Refl
                   (L1 _, _, R1 _) -> Refl
```

```haskell
defaultLeqTransitive
  :: (Generic a, VOrd (Rep a))
  => Π (x, y, z :: a)  -> (x <= y) :~: True
  -> (y <= z) :~: True -> (x <= z) :~: True
defaultLeqTransitive = ???
```

```haskell
defaultLeqTransitive
  :: (Generic a, VOrd (Rep a))
  => Π (x, y, z :: a)  -> (x <= y) :~: True
  -> (y <= z) :~: True -> (x <= z) :~: True
defaultLeqTransitive x y z xLeqY yLeqZ =
  leqTransitive (from x) (from y) (from z)
                xLeqY yLeqZ
```

```haskell
defaultLeqTransitive
  :: (Generic a, VOrd (Rep a))
  => Π (x, y, z :: a)  -> (x <= y) :~: True
  -> (y <= z) :~: True -> (x <= z) :~: True
defaultLeqTransitive x y z xLeqY yLeqZ =
  leqTransitive (from x) (from y) (from z)
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```haskell
defaultLeqTransitive
  :: (Generic a, VOrd (Rep a))
  => Π (x, y, z :: a)  -> (x <= y) :~: True
  -> (y <= z) :~: True -> (x <= z) :~: True
defaultLeqTransitive x y z xLeqY yLeqZ =
  leqTransitive (from x) (from y) (from z)
                xLeqY yLeqZ
```

```
Expected type: (from x <= from y) :~: True
Actual type:   (x <= y) :~: True
```

```
defaultLeqTransitive
  :: (Generic a, VOrd (Rep a))
  => Π (x, y, z :: a)  -> (x <= y) :~: True
  -> (y <= z) :~: True -> (x <= z) :~: True
defaultLeqTransitive x y z xLeqY yLeqZ =
  leqTransitive (from x) (from y) (from z)
                    xLeqY yLeqZ
```

```
Expected type: (from x <= from y) :~: True
Actual type:   (x <= y) :~: True
```

```
class (Generic a, Ord a, Ord (Rep a))
 => GOrd a where
  genericLeqC
    :: Π (x, y :: a)
    -> (x <= y) :~: (defaultLeq x y)
```

```haskell
class (Generic a, Ord a, Ord (Rep a))
  => GOrd a where
   genericLeqC
     :: Π (x, y :: a)
     -> (x <= y) :~: (defaultLeq x y)
```

```haskell
defaultLeq :: (Generic a, Ord (Rep a))
           => a -> a -> Bool
defaultLeq x y = (from x <= from y)
```

```
defaultLeqTransitive
  :: (Generic a, VOrd (Rep a))
  => Π (x, y, z :: a)  -> (x <= y) :~: True
  -> (y <= z) :~: True -> (x <= z) :~: True
defaultLeqTransitive x y z xLeqY yLeqZ =
  leqTransitive (from x) (from y) (from z)
                xLeqY yLeqZ
```

```haskell
defaultLeqTransitive
  :: (Generic a, VOrd (Rep a), GOrd a)
  => Π (x, y, z :: a)  -> (x <= y) :~: True
  -> (y <= z) :~: True -> (x <= z) :~: True
defaultLeqTransitive x y z xLeqY yLeqZ =
  leqTransitive (from x) (from y) (from z)
                xLeqY yLeqZ
```

```haskell
defaultLeqTransitive
  :: (Generic a, VOrd (Rep a), GOrd a)
  => Π (x, y, z :: a)  -> (x <= y) :~: True
  -> (y <= z) :~: True -> (x <= z) :~: True
defaultLeqTransitive x y z xLeqY yLeqZ
  | Refl <- genericLeqC x y
  , Refl <- genericLeqC y z
  , Refl <- genericLeqC x z
  = leqTransitive (from x) (from y) (from z)
                  xLeqY yLeqZ
```

```
defaultLeqTransitive
```

```
Expected type: (from x <= from y) :~: True
Actual type:    (x <= y) :~: True
```

```
  -> (y <= z) :~: True -> (x <= z) :~: True
defaultLeqTransitive x y z xLeqY yLeqZ
  | Refl <- genericLeqC x y
  , Refl <- genericLeqC y z
  , Refl <- genericLeqC x z
  = leqTransitive (from x) (from y) (from z)
                  xLeqY yLeqZ
```

```haskell
defaultLeqTransitive
  :: (Generic a, VOrd (Rep a), GOrd a)
  => Π (x, y, z :: a)  -> (x <= y) :~: True
  -> (y <= z) :~: True -> (x <= z) :~: True
defaultLeqTransitive x y z xLeqY yLeqZ
  | Refl <- genericLeqC x y
  , Refl <- genericLeqC y z
  , Refl <- genericLeqC x z
  = leqTransitive (from x) (from y) (from z)
                  xLeqY yLeqZ
```

```haskell
defaultLeqTransitive
  :: (Generic a, VOrd (Rep a), GOrd a)
  => Π (x, y, z :: a)  -> (x <= y) :~: True
  -> (y <= z) :~: True -> (x <= z) :~: True
defaultLeqTransitive x y z xLeqY yLeqZ
  | Refl <- genericLeqC x y
  , Refl <- genericLeqC y z
  , Refl <- genericLeqC x z
  = leqTransitive (from x) (from y) (from z)
                  xLeqY yLeqZ
```

```
defaultLeqTransitive
  :: (Generic a, VOrd (Rep a), GOrd a)
  => Π (x, y, z :: a)  -> (x <= y) :~: True
  -> (y <= z) :~: True -> (x <= z) :~: True
defaultLeqTransitive x y z xLeqY yLeqZ
  | Refl <- genericLeqC x y
  , Refl <- genericLeqC y z
  , Refl <- genericLeqC x z
  = leqTransitive (from x) (from y) (from z)
                  xLeqY yLeqZ
    :: (from x <= from z) :~: True
```

```
data T a = MkT1 a | ... | MkTn a
```

```
data T a = MkT1 a | ... | MkTn a
  deriving Generic
```

```haskell
data T a = MkT1 a | ... | MkTn a
  deriving Generic
instance Ord a => Ord (T a) where
  (<=) = defaultLeq
```

```haskell
data T a = MkT1 a | ... | MkTn a
  deriving Generic
instance Ord a => Ord (T a) where
  (<=) = defaultLeq
instance GOrd a => GOrd (T a) where
  genericLeqC _ _ = Refl
```

```haskell
data T a = MkT1 a | ... | MkTn a
  deriving Generic
instance Ord a => Ord (T a) where
  (<=) = defaultLeq
instance GOrd a => GOrd (T a) where
  genericLeqC _ _ = Refl
instance VOrd a => VOrd (T a) where
  leqTransitive = defaultLeqTransitive
```

```haskell
data T a = MkT1 a | ... | MkTn a
  deriving Generic
instance Ord a => Ord (T a) where ?
  (<=) = handwrittenLeqImpl
instance GOrd a => GOrd (T a) where
  genericLeqC _ _ = Refl
instance VOrd a => VOrd (T a) where
  leqTransitive = defaultLeqTransitive
```

```haskell
data T a = MkT1 a | ... | MkTn a
  deriving Generic
instance Ord a => Ord (T a) where
  (<=) = handwrittenLeqImpl
instance GOrd a => GOrd (T a) where
  genericLeqC = handwrittenAndDefaultCoincide
instance VOrd a => VOrd (T a) where
  leqTransitive = defaultLeqTransitive
```

?

More! (In the paper)

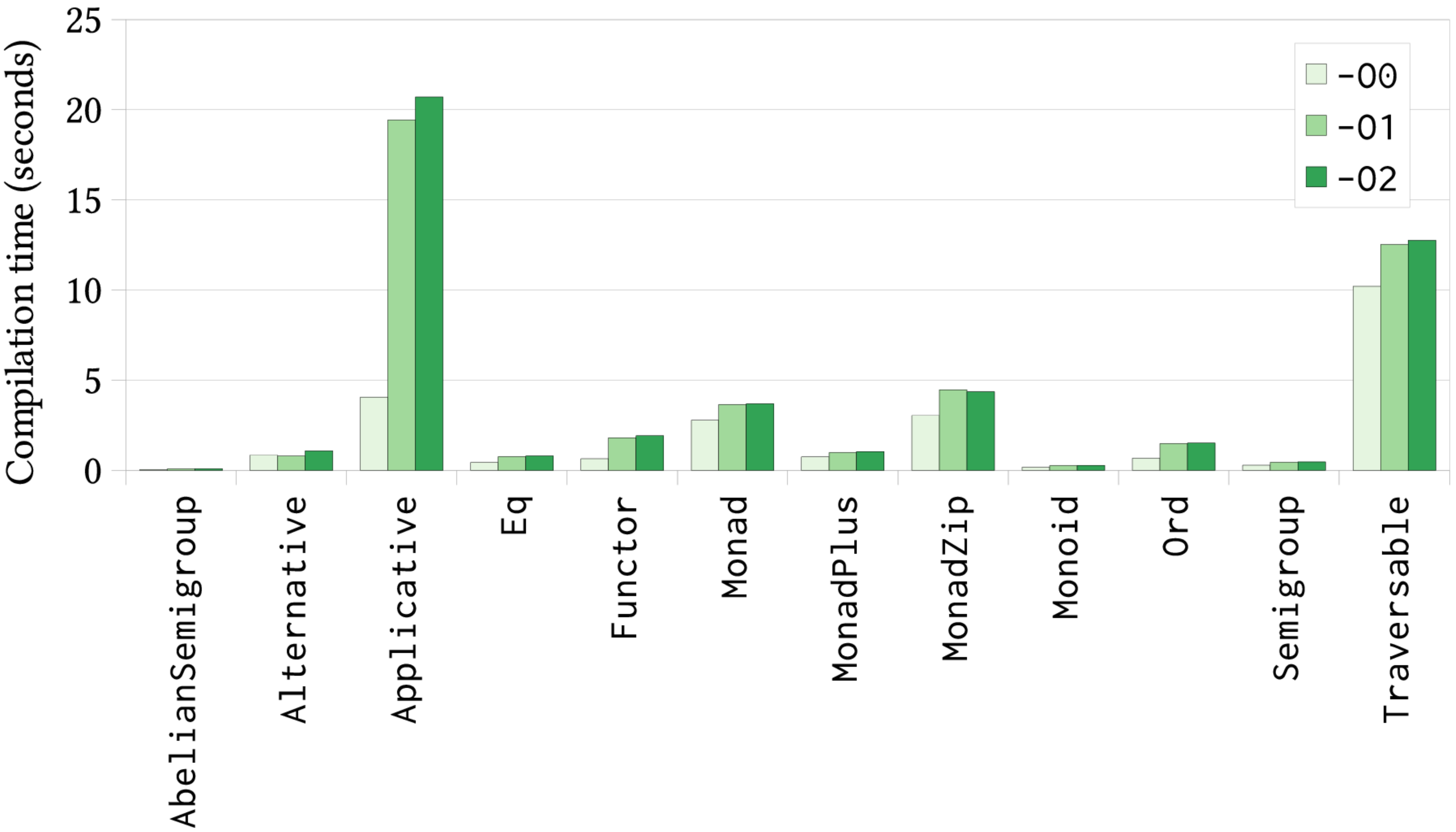
- Generically verifiying more laws from classes in base (`Eq`, `Functor`, `Monad`, `Traversable`, etc.)
- Porting over ideas to other dependently typed languages
  - Coq (successful!)
  - LiquidHaskell (not yet successful…)
- Compile times
- Comparison to other generic programming techniques in dependent types (e.g., univalent transport from HoTT)

verified-classes

- Scrap your type class proof boilerplate as easily as any other type class boilerplate
- Flexible enough to deal with existing code
- Implemented in GHC, but ideas can be ported to other dependently typed languages

`https://gitlab.com/RyanGlScott/verified-classes`

```
Theorem defaultLeqReflexive :
  forall {a : Type} `{VGeneric a}
         `{VOrd (Rep a)} `{Ord a}
         `{! GOrd a}
         (x : a), leq x x = True.
Proof.
  intros. rewrite genericLeqC.
  unfold genericLeq.
  apply leqReflexive.
Qed.
```



Compilation time (seconds)
25
20
15
10
5
0
-O0
-O1
-O2
AbelianSemigroup
Alternative
Applicative
Eq
Functor
Monad
MonadPlus
MonadZip
Monoid
Ord
Semigroup
Traversable