The `verified-classes` Library: Big Proofs, Little Tedium

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```haskell
class Ord a where
  (<=) :: a -> a -> Bool
```

```haskell
-- Transitivity:
--   if x <= y && y <= z,
--   then x <= z
class Ord a where
  (<=) :: a -> a -> Bool
```

```haskell
class Ord a => VOrd a where
  leqTransitive
    :: Π (x, y, z :: a)
    -> (x <= y) :~: True
    -> (y <= z) :~: True
    -> (x <= z) :~: True
```

```
data T a = MkT1 a | MkT2 a | MkT3 a
```

```haskell
data T a = MkT1 a | MkT2 a | MkT3 a
```

```haskell
instance VOrd a => VOrd (T a) where
  leqTransitive t t' t'' Refl Refl =
    case (t, t', t'') of
      (MkT1 x, MkT1 y, MkT1 z)
        | Refl <- leqTransitive x y z Refl Refl
        = Refl
      (MkT2 x, MkT2 y, MkT2 z)
        | Refl <- leqTransitive x y z Refl Refl
        = Refl
      (MkT3 x, MkT3 y, MkT3 z)
        | Refl <- leqTransitive x y z Refl Refl
        = Refl
      (MkT1 _, _, MkT2 _)
        = Refl
      (MkT1 _, _, MkT3 _)
        = Refl
      (MkT2 _, _, MkT3 _)
        = Refl
```

```haskell
data T a = MkT1 a | MkT2 a | MkT3 a
```

```haskell
instan    Ord a => VOrd (T a) where
  le      ive t t' t'' Refl Refl
    c      ', t'') of
      (     kT1 y, MkT1 z)
        |      leqTransit          Refl Refl
        = R
      (MkT2 x          M
        | Refl                 x y z Refl Refl
        = Refl
      (MkT3 x, MkT          z)
        | Refl <           e x y z Refl Refl
        = Refl
      (MkT1              _)
        = R
      (M          MkT3 _)

             _, MkT3 _)
            fl
```

```haskell
data T a = MkT1 a | MkT2 a | MkT3 a

instance VOrd a => VOrd (T a) where
  leqTransitive = defaultLeqTransitive
```

```
data T a = MkT1 a |  ...   | MkTn a

instance VOrd a => VOrd (T a) where
  leqTransitive = defaultLeqTransitive
```

Generic and Flexible Defaults for Verified, Law-Abiding Type-Class Instances

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Abstract

Dependently typed languages allow programmers to state and prove type class laws by simply encoding the laws as class methods. But writing implementations of these methods frequently give way to large amounts of routine, boilerplate code, and depending on the law involved, the size of these proofs can grow superlinearly with the size of the datatypes involved.

We present a technique for automating away large swaths of this boilerplate by leveraging datatype-generic programming. We observe that any algebraic data type has an equivalent *representation type* that is composed of simpler, smaller types that are simpler to prove theorems over. By constructing an isomorphism between a datatype and its represen-

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1 Introduction

Various programming languages support combining type classes [34], or similar features, with dependent type systems, including Agda [11], Clean [29], Coq [25], F* [19], Idris [7], Isabelle [14], and Lean [5]. Even Haskell, the language which inspired the development of type classes, is moving towards adding full-spectrum dependent types [12, 35], and determined Haskell users can already write many dependently typed programs using the *singletons* encoding [13].

Type classes and dependent types together make an appealing combination since many classes come equipped with

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We present a technique for automating away large swaths of this boilerplate by leveraging datatype-generic programming. We observe that any algebraic data type has an equivalent *representation type* that is composed of simpler, smaller types that are simpler to prove theorems over. By constructing an isomorphism between a datatype and its represen-

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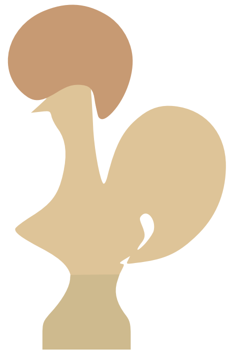

Hackage :: [Package]

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base: Basic libraries

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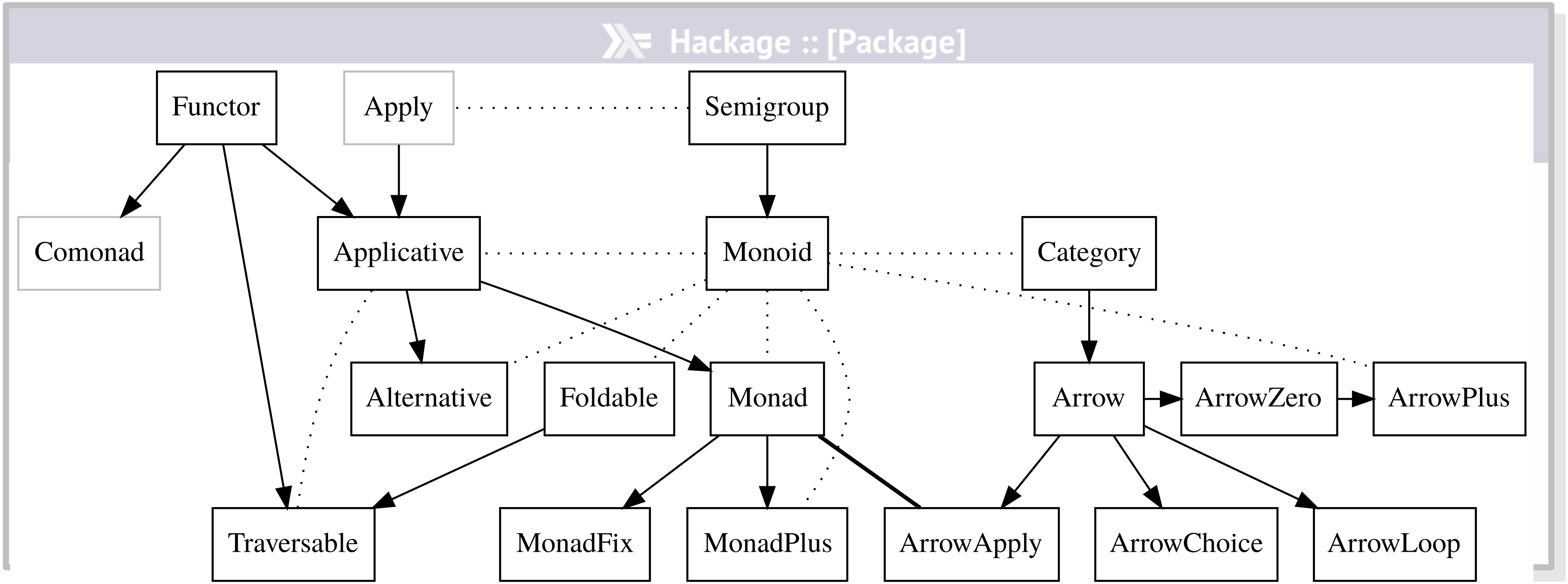
This package contains the Standard Haskell Prelude and its support libraries, and a large collection of useful libraries ranging from data structures to parsing combinators and debugging utilities.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MonadFix
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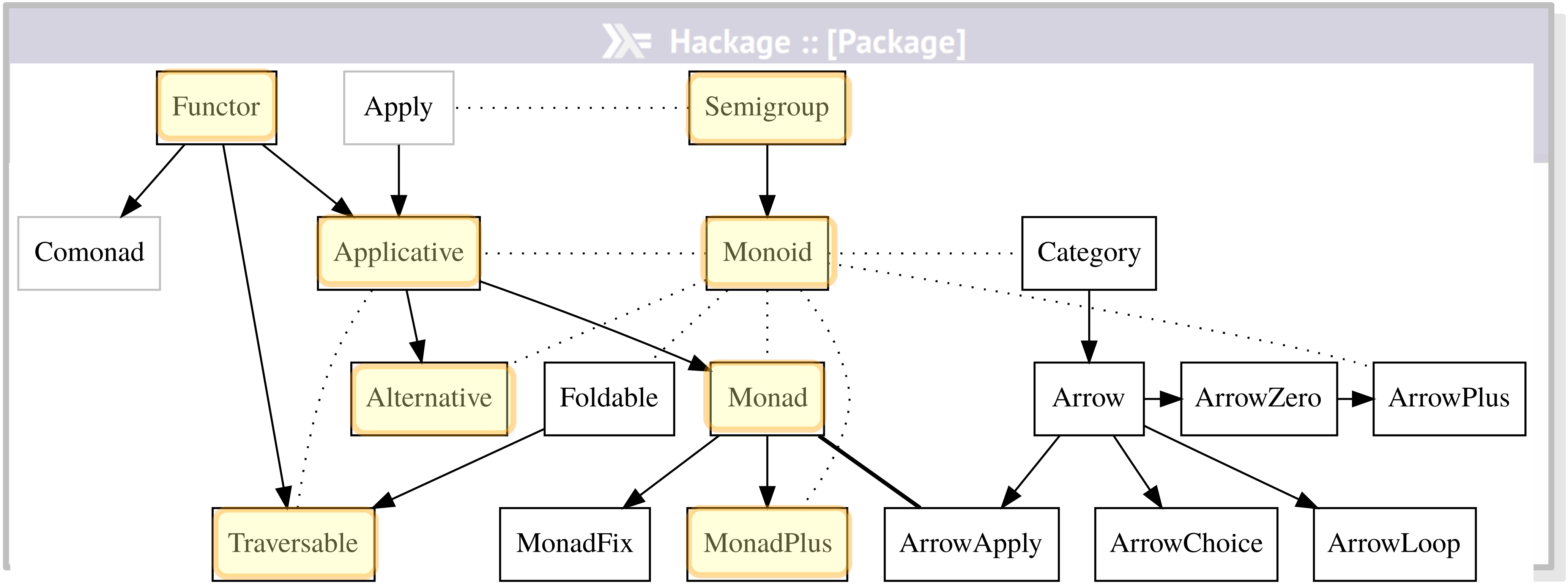
ArrowChoice
ArrowLoop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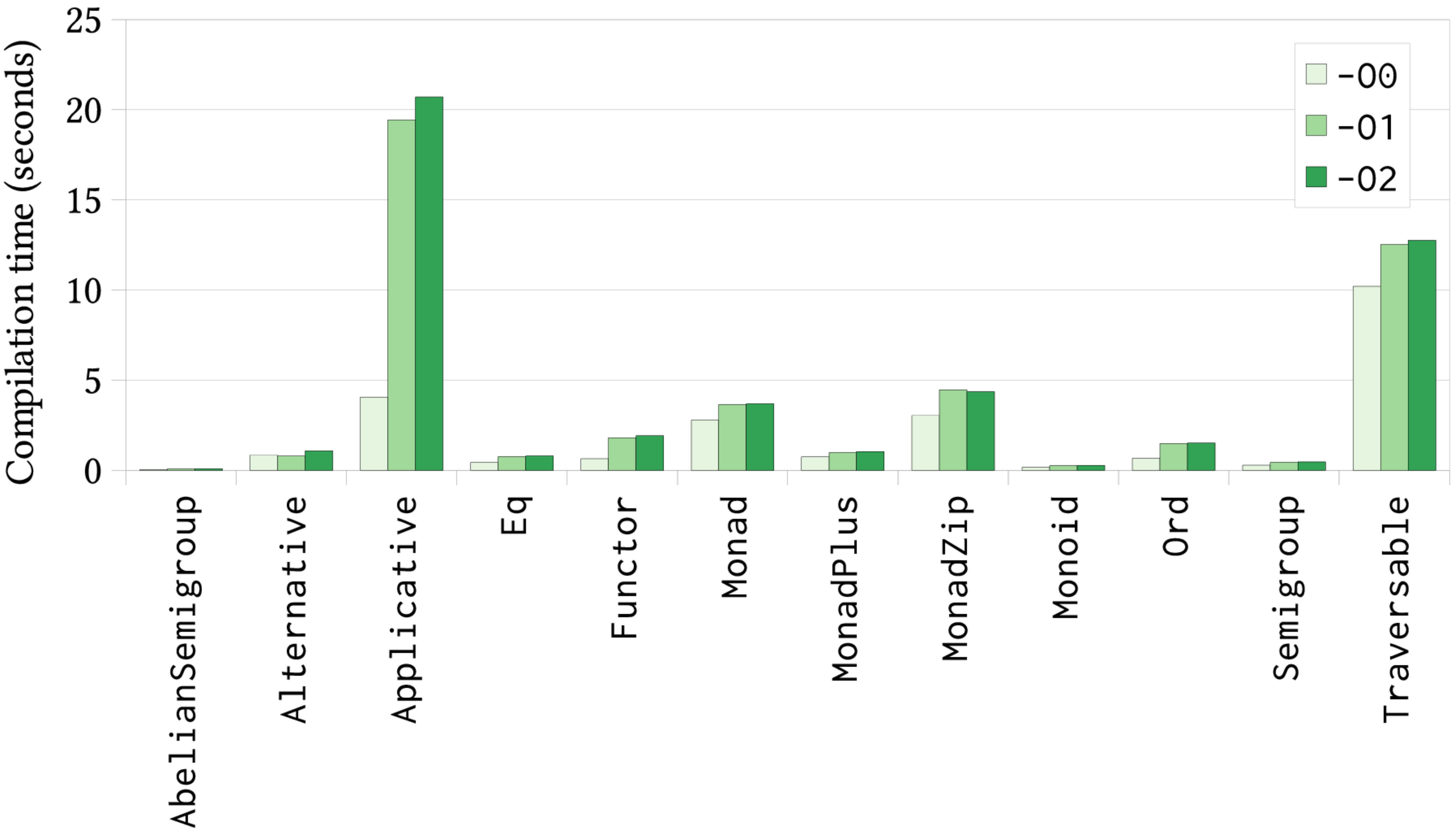

Compilation time (seconds)
25
20
15
10
5
0
-O0
-O1
-O2
AbelianSemigroup
Alternative
Applicative
Eq
Functor
Monad
MonadPlus
MonadZip
Monoid
Ord
Semigroup
Traversable

verified-classes

- Scrap your type class proof boilerplate as easily as any other type class boilerplate
- Flexible enough to deal with existing code
- Implemented in GHC, but ideas can be ported to other dependently typed languages

https://gitlab.com/RyanGlScott/verified-classes